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Equity Options in India: An Empirical Examination

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1 Introduction

It is obvious to even the most naïve investor that a right to buy at 90 what is currently trading at 100 is worth 10. It takes a little more thought to realise that the value of this right is actually more than 10 because while there is a right to buy, there is no obligation to buy. When the time comes to exercise the right, the investor can look at the then prevailing asset price and exercise the right only if the asset price is more than 90. This downside protection is valuable and option markets are all about putting a value on this downside protection.

In early stages of the development of a market, it is possible that this downside protection is ignored (not valued at all) or under-valued. Of course, the market is a great teacher and those who are naïve enough to ignore the value of downside protection will soon learn about it at their cost. Those who learn too slowly will perhaps quit the market after suffering heavy losses. Therefore, in mature markets, one would expect downside protection to be priced reasonably well. This need not be so in the nascent stages of the market.

What has been referred to above as the pricing of downside protection is more technically known as the pricing of volatility. The likelihood that

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an asset now trading at 100 will trade below 90 a month from now depends critically on how volatile its price is. Option markets can thus be seen as markets for volatility. They are markets in which people make assessments about volatility and then buy and sell volatility in accordance with their assessments. The most important measure of the efficiency of an options market, therefore, is the extent to which it prices volatility correctly.

At one extreme would be a market in which the volatility implied by market prices of options accurately reflects the actual volatility (or more precisely the expected future volatility). At the other extreme would be a market in which volatility is not priced at all and the downside protection is completely ignored.

Markets in which volatility is totally ignored are not hypothetical. The Indian convertible bond market in early 1980s did exhibit this phenomenon quite pervasively. In those days there was a ceiling on the rate of interest that companies could pay on their bonds (debentures) and this ceiling was well below the market rate of interest. To get around the ceiling, companies used convertible bonds where bondholders could convert the bonds into the shares of the company at fixed conversion prices set well below the ruling market price. The expected gains on conversion compensated investors for the below market interest rate. When one looked at this market in the mid 1980s one found that convertible bonds were priced ignoring the downside protection completely. Worse, the right to buy a share at 90 in the future when the share was currently trading at 100 was often valued at *less than* 10. It was as if investors expected the share price to actually decline by the time conversion arrived. One could explain this phenomenon at the time of issue on the assumption that companies rigged their share prices at the time of issuing convertibles to make them appear attractive. But since the phenomenon persisted several months after issue, it could only be described as acute mispricing.

Certainly the Indian markets have come a long way since the early 1980s. In the early 1990s, Indian companies were allowed to issue convertibles in global markets, and they were pleasantly surprised to find that foreign investors placed a positive value on the right to convert into shares at a price above the market price at the time of issue. In the domestic market too, convertible bond and warrant issues since the mid-1990s have received far better valuations than in the 1980s. After 2000, the introduction of the derivatives markets has been accompanied by significant skill enhancement and investor training activities carried out by the exchanges and by market participants. All of this would suggest that volatility pricing in the Indian derivatives market would approach the level of efficiency observed in mature derivatives markets.

This paper seeks to evaluate the efficiency of volatility pricing in the Indian market. To do so, it is necessary to define the benchmark of efficient pricing. The benchmark that we use is Black's variant of the well known Black–Scholes model corrected for its most important bias. The Black/Black–Scholes model has been observed to misprice away-from-the-money options because the actual distribution of stock prices deviates from the log normal distribution assumed in the model. It is necessary to correct this bias by using a volatility smile to modify the volatilities for away-from-the-money options. We therefore use a smile-adjusted Black model. The Black/Black–Scholes model requires an estimate of the volatility. GARCH models are well established for this purpose, and so we incorporate a GARCH model for estimating the implied volatility. The smile-adjusted GARCH-volatility Black model is our benchmark for sophisticated volatility pricing.

At the opposite end to this benchmark is a naïve model that ignores volatility completely and sets the option pricing equal to the intrinsic value of the option. In its pure form, this naïve model is not a model to be taken seriously. Certainly the Indian market has progressed far enough to realize that the downside protection provided by the options is valuable. What we are looking for rather is evidence that this naïve model has not been exorcised completely. For example, it is possible that investors may recognize that they need to add something to the intrinsic value. But if they are not fully sensitive to the complex way in which the value of downside protection depends on the option characteristics, they may well end up with a price that looks very much like adding a constant to the intrinsic value. Going further down the road, one could imagine that investors correctly appreciate the determinants of the option value but under estimate the magnitude of this value. In other words, they may move from the naïve model towards the sophisticated benchmark model, but they may not travel the whole distance. The market price would then look like some kind of weighted average of the naïve model and the sophisticated benchmark model. In other words, the market could be anywhere along the whole spectrum from the naïve to the sophisticated model.

This paper studies the pricing of volatility in the Indian index options market using closing Nifty futures and options prices from June 2001 to February 2002.[2] First, we examine whether the market prices can be explained by our sophisticated benchmark - the smile-adjusted GARCH-volatility Black model. Rather than impose an *a priori* notion of what the smile should be, we fit a smile to the data and examine whether the observed smile is reasonable. Specifically, we employ the Black formula to calculate the implied volatility for each option on each day, and then fit a volatility smile to these implied volatilities.

Next, we use the Breeden-Litzenberger formula to compute the implied probability distribution for the terminal stock index price from the fitted smile. The implied probability distribution is then compared to theoretical models and to the historical distribution to determine whether the observed smile is a reasonable one. For example, we look for evidence that the market is underestimating the probability of market movements in either direction. Such a phenomenon would be indicative of under-pricing of volatility.

We then proceed to investigate how well the naïve model and its more realistic variants perform in explaining the observed market prices. We find that the observed prices are rather close to the average of (a) the intrinsic value of the option (pure naïve model) and (b) the Black value unadjusted for a smile. Crudely, one could say that the Indian market lies almost exactly half way between the naïve world where volatility is ignored and a more sophisticated world where volatility is reasonably priced. Surprisingly, we find little evidence that the market has been moving in the direction of greater sophistication in the pricing of efficiency.

Before proceeding to the details of the empirical estimation, we would like to point out that our work completely side-steps the best known and most talked about mispricing in the Indian derivatives market - the negative cost of carry phenomenon in which the futures trades at a discount to the underlying. In our view this mispricing is partly explained by the short sale

[2] The use of daily closing prices has two severe limitations. First, there is the problem of non synchronicity of prices. The closing option price and closing futures price may not be prices that prevailed simultaneously because the last option trade might have been early in the day while the last futures trade is close to end of day. This is discussed further below. Second, as pointed out by an anonymous referee, the closing prices are based on averages over the last 30 minutes of trading. For both of these reasons, it would be desirable for future research to make use of high frequency price data.

restrictions in the cash market that precludes the reverse cash and carry arbitrage that would normally eliminate this mispricing. Globally, also, it has been observed that futures trade below fair value (though not usually below underlying) in the presence of acute short sale restrictions.[3]

This paper circumvents the issue of the cost of carry completely by looking at the relationship between futures and options prices (given by the Black formula (Eqn. 3.1–3.4) below rather than the Black-Scholes formula, (Eqn. 3.5–3.8) below). In this relationship, the cost of carry is almost irrelevant. The option mispricing identified in this paper is thus completely independent of the cost-of-carry mispricing.

2 Implied Volatilities

The Black formula for call and put options in terms of futures prices is as follows:

$$c = e^{-rt}[FN(d_1) - XN(d_2)] \tag{3.1}$$

$$p = e^{-rt}[XN(-d_2) - FN(-d_1)] \tag{3.2}$$

$$d_1 = \frac{1n(F/X) + \sigma^2 t/2}{\sigma\sqrt{t}} \tag{3.3}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{3.4}$$

where c and p are the call and put prices, r is the risk free interest rate, t is the time to expiry, F is the futures price for the same maturity, N denotes

[3] It is sometimes argued that index arbitrage is not affected by short sale restrictions as a significant amount of index arbitrage can be and is done by institutions who own the stock. Neal (1996) is often cited in support of this proposition. In the body of his paper, however, Neal is more circumspect: “For the period analysed in this study, both the presence of arbitrageurs who are long in stocks and the possible circumvention of short sale restrictions suggest that the downtick rule had little effect on the mispricing. This result may not however extend to other periods. If short sale restrictions are binding, the restrictions will affect the mispricing unless institutions possess sufficient capital to fully exploit negative mispricing arbitrage opportunities.” Jiang, Fung and Cheng (2001) review studies covering Finland, Germany, United Kingdom and Hong Kong that show that short sale restrictions do have an impact on the market. They also show that the lifting of short sale restrictions in Hong Kong after 1994 enhanced the dynamic efficiency of the relationship between the cash and futures markets.

the cumulative normal distribution function and σ is the volatility of the futures price. These formulas may be compared with the Black-Scholes formulas:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \tag{3.5}$$

$$p = Xe^{-rt}N(-d_2) - SN(-d_1) \tag{3.6}$$

$$d_1 = \frac{1n(S/X) + rt + \sigma^2 t/2}{\sigma\sqrt{t}} \tag{3.7}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{3.8}$$

It may be seen that the Black prices are exactly what one would obtain if we used the Black Scholes formula with the stock price S replaced by $e^{-rt}F$. This equivalence is useful if we want to get Black formula prices using a Black Scholes options calculator.

It will be noted that the risk free rate has a very small impact on the option price in the Black formula because it appears in a discounting factor that is applied after taking the difference of the two terms inside the bracket. Even setting it to zero will make a difference of less than one percent for most option prices. In the Black Scholes formula on the other hand, the discounting factor is applied to one of the terms before the subtraction, and the risk free rate makes a huge difference to option values. Put differently, in the Black-Scholes formula, the risk free rate determines whether and how much the option is in or out of money, but this does not happen in Black formula.[4]

Since the results are not sensitive to the risk free rate while using the Black formula, we have used a constant risk free rate of 9 percent to compute option prices and implied volatilities from the Black formula. (The results presented below were re-estimated using risk free rates of 0 percent and 100 percent. Qualitatively, the results were quite similar.)

[4]Often, an option is said to be in or out of the money according as the strike is below or above the current stock price, and if the option is in the money, the difference between the stock price and the strike is often said to be the intrinsic value of the money. This is not quite correct. If we let the volatility go to zero in Eqn. (3.1)–(3.8) above, we get the intrinsic value of a call as e^{-rT} max(0,$F-X$) or max(0,$S-e^{-rT}X$). Thus the moneyness of the option and its intrinsic value are related to $F - X$ and not $S - X$. (in the case of a put option, the intrinsic value is related to $X - F$. This is the viewpoint adopted in this paper).

For about 6.5 percent of all calls and about 7.5 percent of all puts, the implied volatility was undefined because the option traded below its intrinsic value. While a few such instances are to be expected because of non synchronous trading[5] while using closing prices, the large percentage of such instances is itself prima facie indicative of mispricing of options. Analysis of the reasons for this is an area for future research. For the purposes of this paper, these options were dropped from the sample and the analysis was conducted using only options for which the implied volatility could be calculated.

3 Volatility Smiles

The volatility smile is the relationship between the implied volatility and the strike price for the same maturity. There is thus a different smile for each maturity on each day. In practice however, it is common to estimate a single smile by relating the implied volatility to the "moneyness" of the option: $\frac{1n(F/X)}{\sqrt{t}}$. (Positive values of the moneyness indicate that a call option is in-the-money and a put option is out of the money.) Note also that there is a *negative* relation between moneyness and strike price for a fixed futures price. For fixed maturity (and futures price), the moneyness is essentially the negative of the logarithm of the strike price. The fact that the futures price is also subsumed into the definition of the moneyness allows us to estimate a single smile for the entire time period under study.

We begin by examining a scatter diagram of the implied volatility against moneyness (Figure 3.1 and 3.2).

A visual inspection reveals the following qualitative features:

- The smiles are V shaped. Typically, the smile for equity options is more like a sneer - a downward sloping curve when plotted against

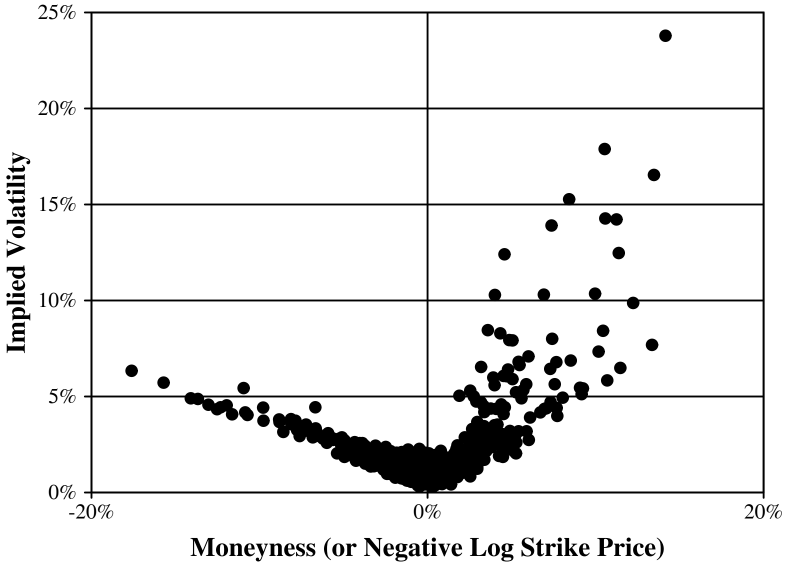
[5]Trading in options was rather thin throughout this period particularly when we recognise that the modest daily trading volume is distributed over put and call options of several different strikes and maturities. This means that the closing price of many options came from trades done several hours before close. Futures prices might also be stale, but perhaps less so. Though futures trading volumes were also quite modest, futures were traded more heavily than options, and the trading volume was distributed over only three maturities. In many cases, therefore, the closing option price and futures price were not prices that prevailed simultaneously. If there are substantial intra-day price movements, an option that was above intrinsic value when it was traded early in the day might appear to be below intrinsic value when the futures closed at a different price later in the day.

the strike price. Smiles in currency options are often U shaped or saucer shaped. The shapes that we observe have some similarity to that observed for currency options.

- The smiles are markedly different for puts and calls. The V is tilted towards the left for calls and tilted towards the right for puts. As already stated, put-call parity requires the two smiles to be the same. Here it is visually evident (even without the statistical tests presented later) that the smiles differ sharply.

Figure 3.1 Call Implied Volatility.

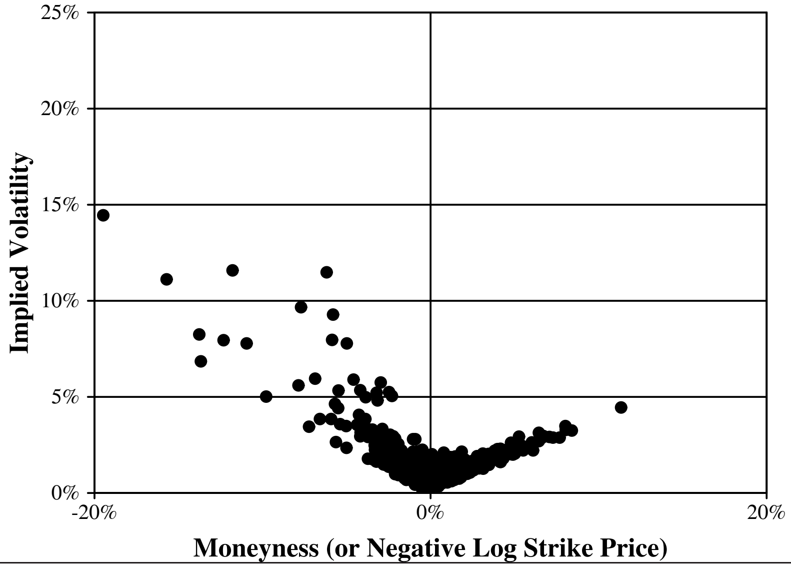
This is a scatter diagram of the implied volatility of call options plotted against the moneyness of the option defined as $ln(F/X)/\sqrt{t}$ where F is the current futures price, X is the exercise or strike price and t is the time to maturity. For a fixed maturity and fixed futures price, the moneyness is essentially the negative log strike price (apart from some additive and multiplicative constants). The data covers the period from June 2001 to February 2002 for options on the NSE Nifty Index.



Before proceeding to statistical analysis of the smile, it is necessary to account for time variation in the implied volatility. Not to do so could potentially vitiate the results because of the well known "omitted variable bias." To model the time variation in implied volatility we use an estimate of historical volatility obtained from the exponentially weighted moving

Figure 3.2 Put Implied Volatility.

This is a scatter diagram of the implied volatility of put options plotted against the moneyness of the option defined as $ln(F/X)/\sqrt{t}$ where F is the current futures price, X is the exercise or strike price and t is the time to maturity. For a fixed maturity and fixed futures price, the moneyness is essentially the negative log strike price (apart from some additive and multiplicative constants). The data covers the period from June 2001 to February 2002 for options on the NSE Nifty Index.



average (EWMA) method which is widely used for this purpose. The exponential weighting coefficient for the EWMA was estimated by Nonlinear Least Squares (NLLS) to provide the best linear fit to the implied volatility. This procedure results in the following regression equation:[6]

$$\begin{array}{rcll} V & = & 0.00654 + 0.51559H & R^2 = 0.07 \\ & & (15.96) \quad\ (18.17) & F(2,4168) = 330.2 \end{array}$$

where V is the implied volatility[7] and H is the historical volatility from an EWMA with the optimal exponential weighting coefficient of 0.83802.

[6]The t statistics are in parentheses.

[7]The volatilities, both historical and implied are expressed on a per day basis (they have not been annualised).

The regression is highly significant[8] though the explanatory power is rather low.

The estimate from this regression is actually a GARCH estimate. To see this, we rewrite the intercept 0.00654 as 0.0134(1-0.51559), and recall that H is a weighted average of yesterday's volatility and today's squared return with weights 0.83802 and (1-0.83802) respectively. We see that the regression estimate is a weighted average of:

- a long run volatility (1.34 percent) with weight (1-0.51559)
- yesterday's volatility with weight 0.51559*0.83802 and
- today's squared return with weight 0.51559*(1-0.83802).

This is therefore a GARCH model except that it is estimated by the best fit not to the actual volatility but to the implied volatility.

We now proceed to model the volatility smile by regressing the implied volatility on H, M^+ and M^- where H is as above, and M^+ and M^- are defined as max(0,M) and max(0,$-M$) respectively and M is the moneyness defined above as $\frac{1n(F/X)}{\sqrt{t}}$. We estimate the regressions for call options, put options and both options together. The estimates are as follows:

Table 3.1 An NLLS regression of the volatility smile.

	M^-	M^+	H	Intercept	R-square	F and df
Call	0.306516	0.776702	0.206408	0.004394	0.71	1968
	-27.89	-73.32	-8.47	-13.01		(4, 2421)
Put	0.602442	0.279282	0.222035	0.005246	0.78	2070
	-68.51	-27.55	-11.15	-19.13		(4, 1741)
Both	0.622162	0.43102	0.179747	0.004494	0.62	2269
	-68.78	-49.1	-9.14	-16.52		(4, 4166)

The equations have high explanatory power and are highly significant. All the coefficients and intercepts are also highly significant. Call and put options have sharply different slopes for M^- and M^+. The F-test for equality of all coefficients in the two regressions (call and put) is highly significant (F=425 with (4, 4166) df). The t-test shows that the differences in slopes for M^- and M^+ between the two regressions are statistically highly significant (t statistics of -21.02 and 33.93 respectively). These tests firmly establish the difference between the smiles for call and put options.

[8] Unless otherwise stated, all significance tests in this paper are at the 0.1% level (p=0.001).

This could also be regarded as evidence of violation of put-call parity, but we must be careful in drawing such an inference.[9]

While the estimated smiles have high explanatory power and are highly significant, some further refinements appear desirable. First, the V-shape that we see in the scatter diagram has a rounded vertex while the linear regression produces a V-shape with a sharp corner. As a result, the smile is non-differentiable when the option is at the money. In the subsequent analysis of the implied probability density, we need to compute the second derivative of the option price with respect to the strike price, and the non-differentiability of the smile is a problem.

We resort to a hyperbola as a simple way to produce a rounded vertex by adding only one extra parameter. The 'V' from the linear regression is a pair of lines $y = -ax$ and that can be represented by the single equation (a degenerate hyperbola) $(y + ax)(y - bx) = 0$ where y denotes the implied volatility and x denotes the moneyness, M. If we add a constant c^2 to the equation, $(y + ax)(y - bx) = c^2$, we get a hyperbola with a rounded vertex that becomes increasingly flatter as c increases. The idea is that the parameter c of the hyperbola can be estimated by non-linear least squares. Solving the quadratic equation of the hyperbola for y gives the expression $y = \frac{-(a-b)x \pm \sqrt{(a+b)^2x^2+4c^2}}{2}$ to be estimated by NLLS. To this must be added the expression involving historical volatility $d + \alpha H$.

[9] To establish violation of put-call parity, it is not sufficient to establish that a fitted model violates put-call parity. It is necessary to show the violation in actual prices. The difference between the two smiles does prove that any valuation model that uses the Black-Scholes model with a volatility smile would violate put-call parity. But it is possible that there are other non Black-Scholes models that provide reasonably good fits to actual prices. We present such a model later in this paper and discuss its implications for put-call parity. One may be tempted to test for put-call parity violation in the actual prices directly by looking at the prices of put and call options with the same strike price. This testing is problematic because of the phenomenon of asynchronous trading described earlier. Put call parity is even more severely affected by asynchronous trading as the parity holds only when the prices of put, call and future are all as at the same instant of time. Any slight non simultaneity in the prices of puts, calls and futures would cause a violation of the put-call parity. The mean absolute violation of put-call parity (in the sample of over 1600 observations where puts and calls of the same strike price and maturity were traded on the same day) is about Rs.2.5 representing nearly 10 percent of the mean put price. A put-call parity violation of at least Rs.5 is found in about 14 percent of the cases. These violations are indeed quite large but it is difficult to determine how much of these are apparent violations due to asynchronous trading and how much is real. A definitive answer on put call parity would perhaps require intra-day prices.

On further visual inspection of the scatter diagram, we observe a faint trace of non-linearity in the arms of the V-shape. This is most evident in the right arm of the V-shape for call options. This suggests adding some quadratic[10] terms to the equation - perhaps, the squares of M^- and M^+. Having just used a hyperbola to get rid of M^- and M^+, we do not want to give up differentiability again by putting them back in. Instead, we add ey^2 to the equation where e is another parameter to be estimated and y is the solution of the quadratic equation for the hyperbola. This technique also economises on parameters since introducing M^-2 and M^+2 into the equation would have added two parameters rather than the one that we have brought in.

Our equation for the implied volatility is therefore:

$$V = d + \alpha H + y + ey^2 \tag{3.9}$$

$$y = \frac{-(a-b)M \pm \sqrt{(a+b)^2M^2 + 4c}}{2} \tag{3.10}$$

involving six parameters - a, b, c, d, e, α - to be estimated simultaneously by NLLS.[11] The results are as follows in Table 3.2.

Table 3.2 A hyperbolic regression of the volatility smile.

	Call	Put	Both
α	0.25615	0.23982	0.25453
A	0.18652	0.64501	0.24592
B	0.418	0.35948	0.3843
C	0.00144	0.00496	0.00282
D	0.00474	0.00162	0.00327
E	26.84434	0	26.84582
R-square	0.75397	0.78985	0.67824
t-stat for e = 0	14.76	0	22.06
t-stat for c = 0	14.27	8.55	15.34

The linear regression estimated earlier is nested within this model ($c = e = 0$) and the hyperbola without a quadratic correction ($e = 0$) is nested

[10] A better fit might be obtained by including higher powers using Chebyshev or other orthogonal polynomials. As shown below, however, the quadratic approximation provides a good fit to the data.

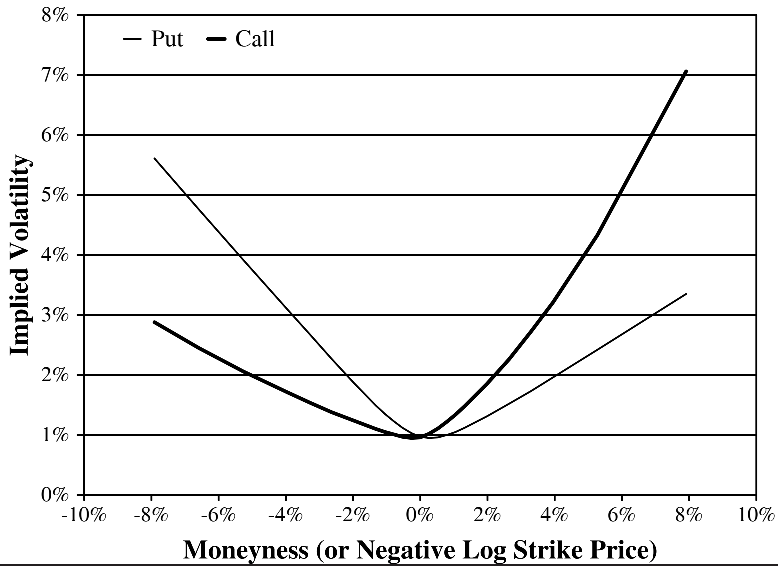
[11] It may be noted that when M is set equal to zero, $y = c$ and the implied volatility (viewed as a regression against H) has an "intercept" $c + d + ec^2$. Thus while the estimated implied volatility subsumes a Garch model, the Garch parameters are not now easy to identify.

between the two. We can therefore use a F test (or equivalently a t test since there is only additional parameter involved) to choose between these nested models. We thus get t tests for the hypotheses $c = 0$ and $e = 0$.

It may be seen that the hyperbolic term c is highly significant in all cases and the quadratic term e is highly significant for call options as well as for all options taken together but not for put options. While one might have expected e to be negative, so that the smile is moderated at high levels, the estimated e is positive so that the smile is actually strengthened at high levels. The estimated smiles are plotted in Figure 3.3.

Figure 3.3 Estimated Smiles.

This is a plot of the fitted smiles for call and put options using a hyperbolic functional form with a quadratic correction. The implied volatility is modelled as a function of the moneyness of the option defined as $ln(F/X)/\sqrt{t}$ where F is the current futures price, X is the exercise or strike price and t is the time to maturity. For a fixed maturity and fixed futures price, the moneyness is essentially the negative log strike price (apart from some additive and multiplicative constants).



4 Implied Probability Density

Breeden and Litzenberger (1978) showed that the price of a primitive security that has a unit payoff when the asset price at time T lies between s and $s + ds$ is given by

$$\left(\frac{\partial^2 c(X,T)}{\partial X^2}\right)_{X=s}$$

where $c(X, T)$ is the price of a call option with strike price X maturing at time T. Multiplying this by e^{rT} gives us the risk neutral probability that the terminal asset price lies between s and $s + ds$. In other words,

$$e^{rT}\left(\frac{\partial^2 c(X,T)}{\partial X^2}\right)_{X=s}$$

gives us the risk neutral probability density of the terminal stock price. By put-call parity, the Breeden-Litzenberger formula holds for put prices as well.[12]

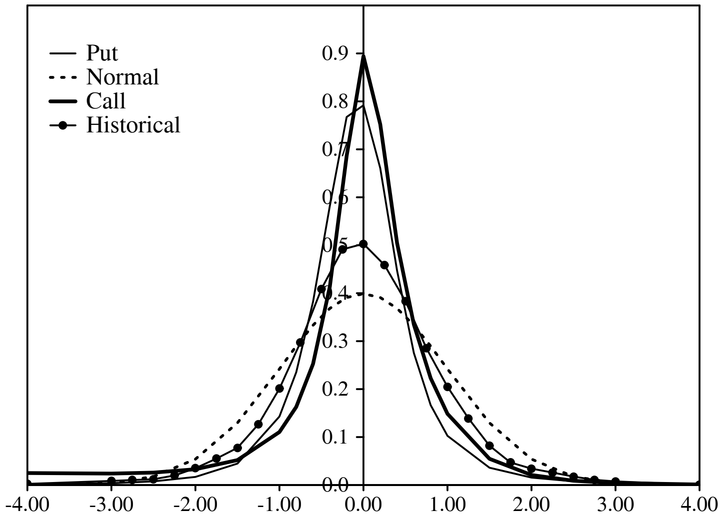
We now derive the risk neutral probabilities implied by the smiles estimated above. For various values of X, we compute the implied volatility using the smile, plug this into the Black-Scholes formula to get the option price. We use numerical differentiation to compute the second derivative of the option price. Since the stock price is approximately lognormally distributed, we transform the computed density to the implied density of the log stock price standardised to a zero mean and unit variance. This is the same as computing the distribution of the logarithmic stock price return. Figure 3.4 plots the computed implied densities from call and put options. For comparison, it also plots the standard normal density as well as the historical probability density of the Nifty index computed in Varma(1999) using data for 1990-1998.

It is seen that the implied densities are much more highly peaked and have thinner tails than the normal density and the historical density (with one exception discussed later below). The high peak and thin tails imply an expectation that the stock price would move within a very narrow range. This is inconsistent with historical experience and any plausible forecast of the future.

[12]Put call parity states that $c = p + f$ where c and p are the call and put prices respectively while f is the price of a position involving the stock or the futures along with some borrowing. This implies that $\frac{\partial^2 c}{\partial X^2} = \frac{\partial^2 p}{\partial X^2} + \frac{\partial^2 f}{\partial X^2} = \frac{\partial^2 p}{\partial X^2}$ since the stock price or futures price is independent of the exercise price.

Figure 3.4 Implied Probability Density of the Log Stock Return.

The implied probability density of the stock price is extracted from the fitted implied volatility smile for put and call options on the NSE Nifty Index using the Breeden-Litzenberger formula. The implied density of the stock price is then converted to a density of the log stock return. This density is compared with the normal density as well with the historical density estimated in Varma (1999) for the period 1990-1999.



The only exception is the fat left tail for call options. This bears some superficial similarity to the findings of Jackwerth and Rubinstein (1996) who observed fat left tails (10 and 100 times fatter than the normal at three and four standard deviations respectively) for index options in the United States after the crash of 1987. This phenomenon of "crashophobia" as Rubinstein called it, made a big difference to the valuation of out-of-the-money put options which should be worth very little under the Black-Scholes model. However, the fat left tail that we see here does not apply to put options. The fat left tail is only for call options, it affects only deep in the money calls, and even here the percentage effect on the price is not very large as these options have large intrinsic values anyway. The fat tail here does not seem to have anything to do with "crashophobia" since we do not see a fail tail for put options.

The only possible explanation for the implied probability distribution is that the market is mispricing volatility quite badly. Specifically, it is severely underestimating the probability of large market movements in either direction. As a result, the probability distribution of stock prices implied by the market prices is totally out of line with theoretical models of the distribution and with the historically observed distribution.

The peak and tails of the implied probability densities would be quite easily explained if we assume that the market price is partly driven by the naïve model. Since the intrinsic value ignores volatility (or alternatively assumes zero volatility), it is clear that the implied distribution from a market which has not exorcised the naïve model would have a large peak at zero (no change in the index) and would have thin tails.

As for the deep-in-the-money calls, a plausible explanation would be that the market is starting with the naïve model (intrinsic value of the option) and adding a small value to it to adjust for volatility. What appears to be happening is that for deep-in-the-money calls where the Black-Scholes value is practically the same as the intrinsic value, the market still adds a small value to the intrinsic value and thus overprices them. This turns up in the analysis as the fat left tail for the call option implied distribution. Why we do not see a corresponding fat right tail for put options is a mystery.

All in all, the implied probability density provides strong evidence that the naïve model is still driving option prices in the Indian market to a significant extent. For more evidence on this, we turn to how well various models fit the actual option prices.

5 Goodness of Fit to Option Prices

While fitting a curve to implied volatilities is regarded as numerically superior to fitting a curve to option prices (Jackwerth, 1999), we are ultimately interested in the goodness of fit to option prices. We therefore use the fitted smiles to calculate option prices from the Black formula and compare the results with the market prices. We compare our results with three other pricing models:

- A ***no smile*** or flat smile model in which the same implied volatility is used for all strikes on the same day.
- A ***constant volatility*** model in which a single long run implied volatility is used for the entire sample.

- A ***intrinsic (or naïve)*** model that assigns the value max(0, $F - X$) to the call option and the value max(0, $X - F$) to the put option.

The goodness of fit is measured by regressing the actual prices on the model prices for each model separately. A good fit would be reflected in a zero intercept, unit slope and high R^2. The regression results are as follows:

Table 3.3 The goodness of fit of alternative volatility models in pricing options.

	Fitted Smile	No Smile	Constant Volatility	Intrinsic Value
Intercept	0.26813	-2.57154	29.43855	10.4149
(t stat)	(-2.57)	(-13.46)	(-39.66)	(-65.96)
Slope	0.99553	0.99479	-0.05063	0.975287
(t stat)	(-412.72)	(-231.85)	(-3.45)	(-232.37)
R^2	0.976	0.928	0.003	0.928
F Statistic df=(1, 4168)	170338	53756	11.93	53995
t-stat for slope = 1	(1.85)	(1.21)	(71.68)	(5.89)

The model based on the fitted smile does extremely well with an R^2 of 0.98. The intercept is not significant at the 0.1 percent level used throughout this paper (it just misses being significant at the one percent level). The slope is not significantly different from unity. The flat smile model has a lower R^2 than the fitted model. Moreover, it has an intercept that is highly significant. The constant volatility model fares disastrously.

Most surprisingly, the intrinsic value model does as well as the no smile model. The regression equation for this model effectively says that the observed prices are rather well explained by simply adding a constant (approximately Rs.10) to the intrinsic value of the option. In other words, the market recognizes that the option is worth more than its intrinsic value, but its adjustment for this is rather well approximated by a constant value of approximately Rs.10. The good performance of this model is strong evidence that the market while moving beyond the naïve model has not given up that model completely.

Another way of examining the role of the naïve model in market pricing is to model market prices as a weighted average of the intrinsic value and the Black prices (with no smile adjustment). A naïve market would be one which places the entire weight on the intrinsic value. As the market becomes more sophisticated, it would place a greater weight on the Black price. An adjustment for fat tails could potentially cause a weight greater than one on the Black price to account for the underpricing of out of the money options by the Black model. To a first approximation, however, a mature market could be assumed to put all the weight on the Black price and put zero weight on the intrinsic value.

The interesting result that emerges is that both the actual prices and the fitted smile prices are quite well approximated by an equally weighted average of the intrinsic value and the no smile prices.[13] The natural interpretation is that the market has moved from the naïve model towards the Black model but has gone only half the way. The regression results are as follows:

$$\begin{aligned}
P_{\text{actual}} &= 2.95425 + 0.50875 P_{\text{intrinsic}} + 0.51701 P_{\text{nosmile}} \qquad R^2 = 0.967 \\
&\quad\ (19.36) \quad\ (69.33) \qquad\qquad (69.07) \qquad F(2, 4167) = 60274 \\
P_{\text{smile}} &= 2.78941 + 0.51260 P_{\text{intrinsic}} + 0.51543 P_{\text{nosmile}} \qquad R^2 = 0.986 \\
&\quad\ (28.27) \quad\ (108.07) \qquad\quad (106.52) \qquad F(2, 4167) = 60274
\end{aligned}$$

Though the fitted smile model is so well approximated by combining the no smile and intrinsic models, it is still true that the fitted smile provides a better fit to the actual prices than the combination.[14] Thus though we can crudely say that the market lies half way between the naïve model and the Black model, there are some other quirks of mispricing that are captured only by the fitted smile. Most important of these would probably be the

[13] This model has implications for put-call parity issue discussed above. The point is that both the no smile model and the intrinsic value model separately satisfy put-call parity. (In fact, the intrinsic value model is a Black-Scholes model with zero volatility.) As such an average of the two would also satisfy the parity condition. Of course, it is true that this average still has a lower R^2 with actual prices than the fitted smile model. But the point is that it might be possible to find another model with a better fit.

[14] We see this by regressing actual prices on the three models - fitted smiles, no smile and intrinsic:

$$\begin{aligned}
P_{\text{actual}} &= 0.60910 + 0.08368 P_{\text{nosmile}} + 0.07780 P_{\text{intrinsic}} + 0.84073 P_{\text{smile}} \\
&\quad\ (4.35) \quad\ (6.90) \qquad\qquad (6.48) \qquad\qquad (41.79) \\
&\quad\ R^2 = 0.976. \quad F(3, 4166) = 57593
\end{aligned}$$

The coefficient of the fitted smile model is not much below unity and is about ten times the coefficients of the other two models. The R^2 is still unchanged up to the third decimal place from a regression on the fitted smile price alone (it increases by 0.0003).

different treatment of puts and calls especially the overpricing of out of the money calls.

The goodness of fit can also be measured in terms of the percentage absolute pricing error, $100 * |\text{actual price} - \text{model price}| \,/\, \text{actual price}$. However, in doing so it is necessary to ignore low price options as even small pricing errors result in large percentage errors for such options. We therefore ignore options whose price is less than one percent of the futures price and compute the mean and median of the percentage absolute pricing error for the remaining options for all models. For comparison, we also present the mean percent error obtained when the model price is replaced by the sample mean of actual prices in Table 3.4.

Table 3.4 Goodness of fit measures for various volatility models using absolute pricing error.

	Percentage Absolute Pricing Error	
Model	Mean (%)	Median (%)
Fitted smile	14.83	10.27
No smile	26.04	11.96
Constant Volatility	76.9	54.13
Intrinsic value + 10[15]	25.84	19.9
Half-way Model + 3[16]	17.84	11.9
Sample Mean Price	50.81	43.8

While the fitted model is superior to the other models, the half-way model (simple average of the intrinsic value and the no smile model) does almost as well.

6 Market Maturity and Learning

One might speculate that as the market becomes more mature and participants learn more about various derivative products and their inter-

[15] The pure naïve model fares very badly as it assigns zero value to all out of the money options. The reasonable R^2 of this model with actual prices is because the regression equation includes an intercept of about 10.41. To give a reasonable chance to the intrinsic model, we simply add a constant 10 to the intrinsic model values. A better solution might be to consider max(10, intrinsic_value).

[16] The half-way model refers to the average of the intrinsic value and the no smile model. The constant of 3 represents the intercept in the regression of actual prices on the intrinsic value and the no smile model.

relationships, price discovery would improve and the observed mispricing would diminish.

Consider the model where the market price is a weighted average of the naïve model and the theoretical price (no smile value). Learning would imply that over time these weights shift away from the naïve model towards the theoretical price. We can test for this by estimating a regression in which the weights (coefficients) vary from month to month. This would require a series of slope dummies for each of the two independent variables (intrinsic value and no smile value). To reduce the number of slope dummies, we rewrite the regression to include a slope dummy only for one independent variable. A regression on intrinsic value and no-smile value is equivalent to a regression on intrinsic value and time value where time value is defined as the excess of the no-smile value over the intrinsic value. In this formulation, it is sufficient to have a series of slope dummies for the time value variable.

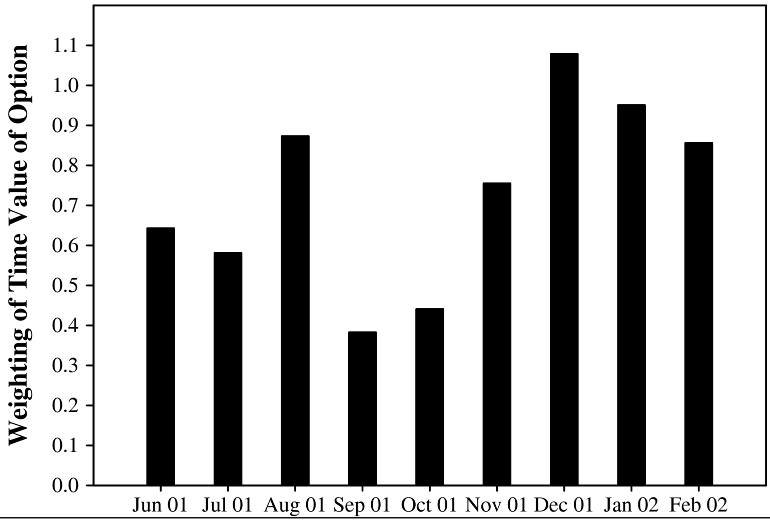
The regression result is as presented in Table 3.5, where we see that all the coefficients are highly significant and there is a great deal of fluctuation from month to month in the coefficient of time value. The accompanying Figure 3.5 provides a visual depiction of this fluctuation. There does not appear to be any clear rising trend in the fluctuations. In fact the last three months of data show a falling trend that is diametrically opposed to a learning hypothesis. The only pattern that is clearly evident is that the coefficient was very low in September and October and has recovered since then.

Table 3.5 Regression coefficients of the weights on instrinsic value vs. time value in option prices.

	Coefficients	t Stat
Intercept	1.08689	8.1572
Intrinsic Value	1.0404	424.3260
Time Value June 2001	0.6430	53.2085
Time Value July 2001	0.5813	48.6956
Time Value August 2001	0.8733	35.5099
Time Value September 2001	0.3830	44.6032
Time Value October 2001	0.4411	40.5385
Time Value November 2001	0.7552	43.6386
Time Value December 2001	1.0789	51.6297
Time Value January 2002	0.9510	45.3899
Time Value February 2002	0.8563	38.0665

Figure 3.5 Month by Month Weighting of Time Value.

The actual option prices are regressed on the intrinsic value of the option and its time value where the time value is the excess of the Black-Scholes price (without any smile adjustment) over the intrinsic value. The coefficient of the time value is allowed to vary from month to month using slope dummies in the regression. The month by month behaviour of this coefficient (the weighting of time value) is plotted above. While the learning hypothesis (market moving towards Black-Scholes pricing over time) would imply a monotonic increase in the coefficient, the above plot shows no clear secular trend.



The most important event in September 2001 was certainly the terrorist attack on the World Trade Centre in the United States on September 11, 2001. There is evidence for a highly significant structural break after that day (the event took place after the Indian markets had closed for the day). We see a sharp spike in historical volatility. More interestingly, there is also a sharp spike in the implied volatility. Specifically, when we include an intercept dummy for the post 9/11 period in the regressions for implied volatility on historical volatility, the coefficient for this dummy is positive and highly significant. The natural interpretation of this result would be that there was an upward shift in the long run volatility in the Garch model after the 9/11 episode.

In this light, the sharply lower coefficients for the Black-Scholes time value during the period following this event is quite disturbing. It appears to suggest that the market prices failed to account correctly for the sharp rise in option values resulting from the spike in implied volatilities following the 9/11 episode.

All in all, the data do not show any clear evidence of learning effects in the markets. Why is this so despite the substantial investments that regulators, exchanges and market participants have made in training and investor awareness? The answer possibly is that market maturity and learning is not just a matter of familiarity with the theory and concepts of option pricing. While some of the learning comes from theory, a significant part comes from "learning-by-doing." More importantly, market maturity requires a transformation of this knowledge into action. This in turn requires substantial investment in organizational systems and processes as well as in information technology. While computer systems can be designed to detect mispricing and arbitrage opportunities by integrating sophisticated pricing models with real-time price feeds, market maturity also requires the ability on the part of market participants to implement trading strategies that exploit these opportunities. It would appear that while the training effort in the derivatives markets have addressed the issue of disseminating the theoretical knowledge about option pricing, the other aspects of market maturity like learning-by-doing and investment in systems and processes will take more time.[17]

It is possible therefore that the learning effects become clearly visible only over a longer time frame of 18-24 months. This is a matter requiring further research using longer time periods.

7 Conclusion

We have established severe mispricing in the Indian index options market even after using futures prices to eliminate the effect of short sale restrictions in the cash market. In particular, volatility is severely underpriced.

We have estimated volatility smiles separately for put and call options and established by statistical significance tests that the smiles are sharply different for calls and puts while put call parity requires that the smiles be the same. The implied probability distribution is more highly peaked and has (except for deep-in-the-money calls) thinner tails than the normal

[17]The author would like to thank an anonymous referee for some of the ideas in this paragraph.

distribution or the historical distribution. The market thus appears to be underestimating the probability of market movements in either direction. At the same time, we see some overpricing of deep-in-the-money calls and some inconclusive evidence of violation of put-call-parity. We also show that the observed prices are rather close to the average of the intrinsic value of the option and its Black-Scholes value (disregarding the smile). Crudely, one could say that the Indian market lies almost exactly half way between the naïve world where volatility is ignored and a more sophisticated world where volatility is reasonably priced. Surprisingly, we find little evidence that the market has been moving in the direction of greater sophistication in the pricing of efficiency. This is an area for future research using high frequency data over longer time periods.

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